Preparing Graph States

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October 11, 2025, ACORN

with **Jim Geelen**, **Donggyu Kim**, **Caleb McFarland**, and **Paul Wollan**

Problem

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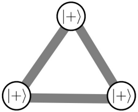
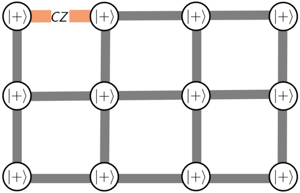


$|H\rangle$



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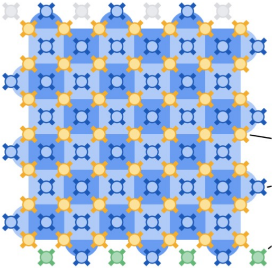
Quantum error correction below the surface code threshold

Google Quantum AI and Collaborators

Nature **638**, 920–926 (2025) | Cite this article



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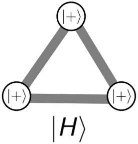

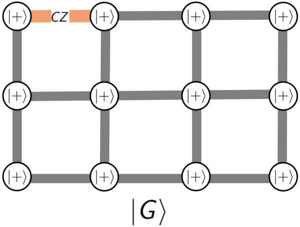

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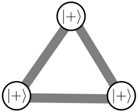

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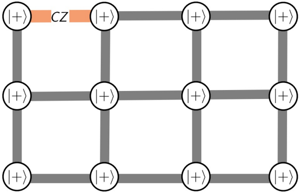

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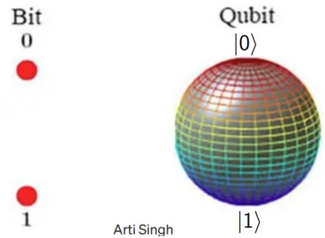

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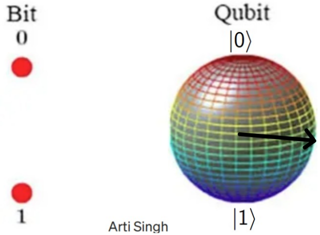

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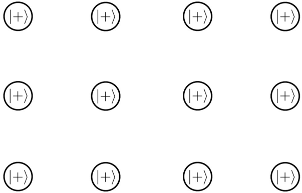

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$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma\\ \delta\end{bmatrix} = \begin{bmatrix}\alpha\gamma\\ \alpha\delta\\ \beta\gamma\\ \beta\delta\end{bmatrix}$$

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$$|\mathbf{01}\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$$

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So $|000\rangle, |001\rangle, |010\rangle, |011\rangle, \ldots, |111\rangle$
form a basis for $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$
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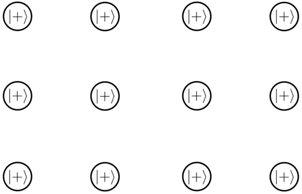

$|G\rangle$

$$|+\rangle \otimes \ldots \otimes |+\rangle = \frac{1}{\sqrt{2}^n} \sum_{s \in \{0,1\}^n} |s\rangle$$

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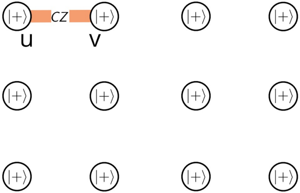

$|G\rangle$

$$CZ_{uv} |+\rangle \otimes \ldots \otimes |+\rangle = CZ_{uv} \frac{1}{\sqrt{2}^n} \sum_{s \in \{0,1\}^n} |s\rangle$$

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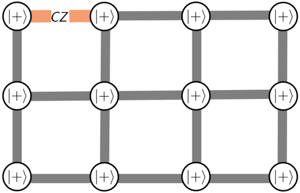

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$$\prod_{uv \in E} CZ_{uv} |+\rangle^{\otimes V}$$

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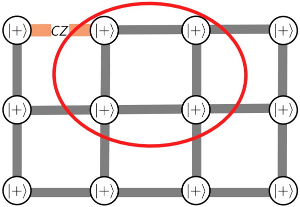

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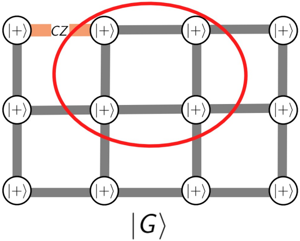

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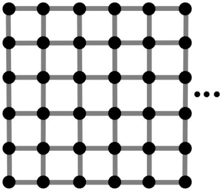

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"A One-Way Quantum Computer"
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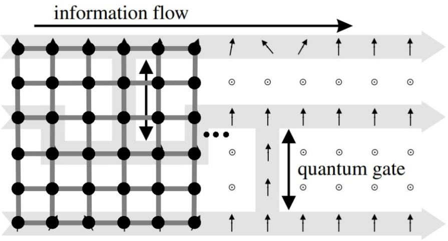

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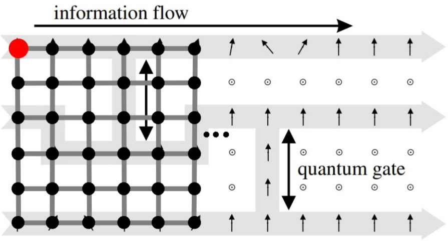

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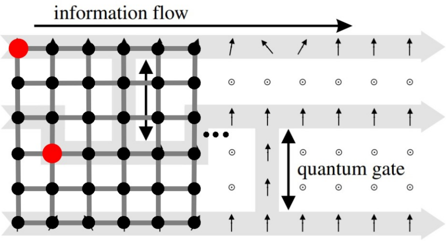

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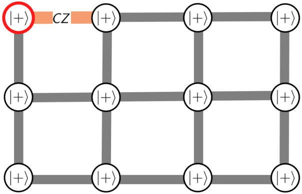

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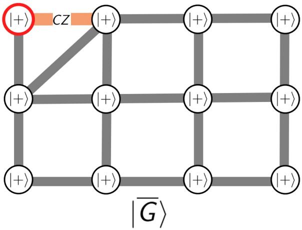

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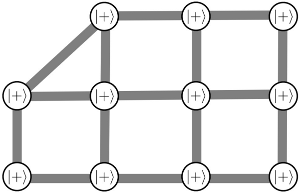

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Theorem (Van den Nest-Dehaene-De Moor 04; Dahlberg-Helsen-Wehner 20)

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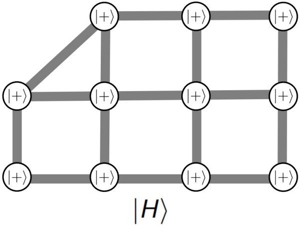

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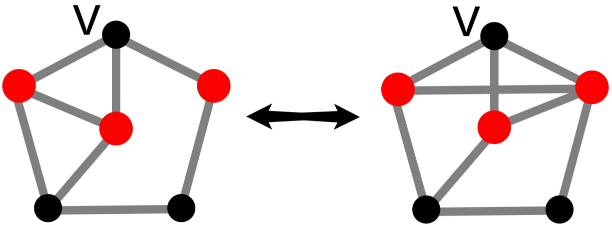

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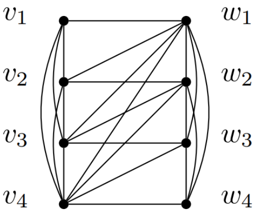
To **locally complement,** select a vertex v and switch adjacencies within the **neighborhood** of v.

Theorem (Kwon-McCarty-Oum-Wollan 21)

If H is a **half-graph***, then any graph with no H-***vertex-minor** *"looks like a shallow tree".*



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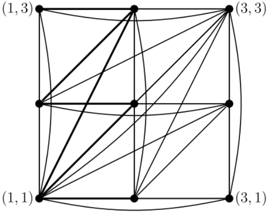

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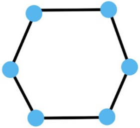
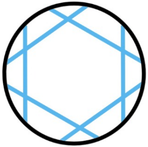
Corollary (Courcelle-Oum; Fomin-Korhonen; Courcelle-Makowsky-Rotics)

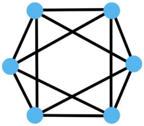
For any **circle graph** *H, we can test if H is a* **vertex-minor** *of an n-vertex graph G in time $\mathcal{O}_H(n^2)$.*



circle graph



chord diagram

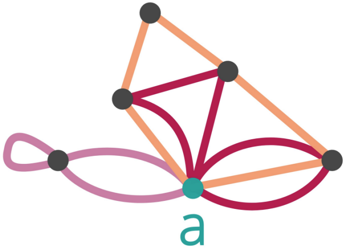


tour graph

Theorem (McCarty 24)

For any Eulerian graph G and vertex **a**, *the* **maximum** *size of a circuit decomposition where every circuit is odd and hits* **a** *equals*

$$\textbf{minimum}_{\gamma', X}\left(\gamma'(E(X)) + \frac{1}{2}|\delta(X)| - odd_{\gamma'}(G - X)\right).$$



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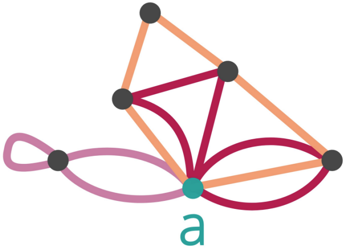

Q: Can we obtain this as a corollary of the Tutte-Berge Formula?

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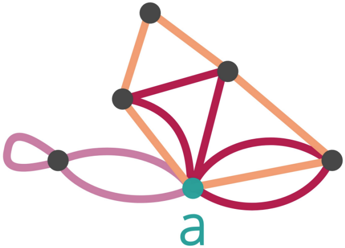

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Q (Claudet): Largest independent set **vertex-minor** in $G(n, 1/2)$?

Thank you!