Vertex-minors and quantum computing

Rose McCarty

Schools of Math and CS

Georgia Tech

October 27, 2025, Bertinoro Workshop on Algorithms and Graphs

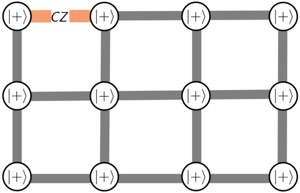
With **Jim Geelen** and **Paul Wollan**.

Quantum Graph States: Bridging Classical Theory and Quantum Innovation—Workshop Summary

Eric Chitambar[1], Kenneth Goodenough[2], Otfried Gühne[3], Rose McCarty[4], Simon Perdrix[5], Vito Scarola*,[6], Shuo Sun[7], and Quntao Zhang[8,9]

$$|G'\rangle = \left(-e^{i\frac{\pi}{2}\frac{X_1-Z_1}{\sqrt{2}}} \otimes Z_2 \otimes Z_3 \otimes e^{i\frac{\pi}{2}\frac{X_4-Z_4}{\sqrt{2}}}\right)|G\rangle.$$

What is the **quantum state** $|G\rangle$ associated with a graph G?



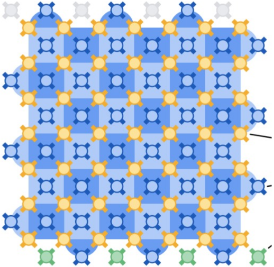
$|G\rangle$

What is the **quantum state** $|G\rangle$ associated with a graph G?

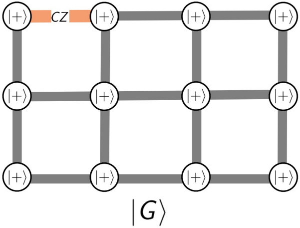
Quantum error correction below the surface code threshold

Google Quantum AI and Collaborators

Nature **638**, 920–926 (2025) | Cite this article

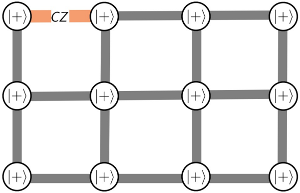


What is the **quantum state** $|G\rangle$ associated with a graph G?



A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

What is the **quantum state** $|G\rangle$ associated with a graph G?

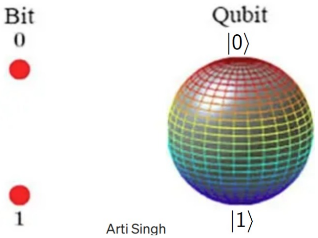


$|G\rangle$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

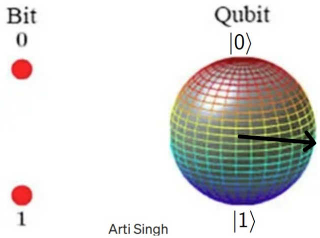
What is the **quantum state** $|G\rangle$ associated with a graph G?



A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1 \\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0 \\ 1\end{smallmatrix}\right]$.

What is the **quantum state** $|G\rangle$ associated with a graph G?

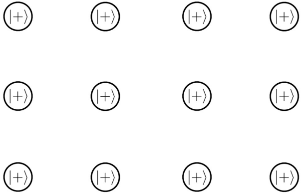


A 1-**qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$|G\rangle$

A 1-**qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma\\ \delta\end{bmatrix} = \begin{bmatrix}\alpha\gamma\\ \alpha\delta\\ \beta\gamma\\ \beta\delta\end{bmatrix}$$

A 1-**qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An *n*-**qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

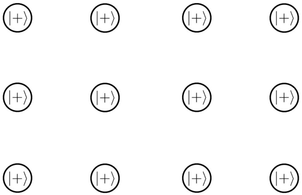
A **1-qubit quantum state** is $\left[\begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.
For example, $|0\rangle = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$.
Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.
An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?

So $|000\rangle, |001\rangle, |010\rangle, |011\rangle, \ldots, |111\rangle$
form a basis for $\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$
called the **computational basis**.

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.
For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.
Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.
An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$|G\rangle$

$$|+\rangle \otimes \ldots \otimes |+\rangle$$

$$=$$

$$\frac{1}{\sqrt{2}^n} \sum_{s \in \{0,1\}^n} |s\rangle$$

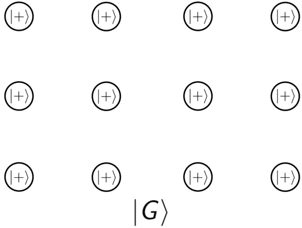
A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1 \\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0 \\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$$|+\rangle^{\otimes V} = \frac{1}{\sqrt{2}^n} \sum_{s\in\{0,1\}^V} |s\rangle$$

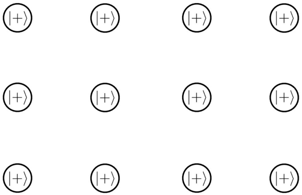
A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$|G\rangle$

$$|+\rangle^{\otimes V} = \frac{1}{\sqrt{2^n}} \sum_{s \in \{0,1\}^V} |s\rangle$$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

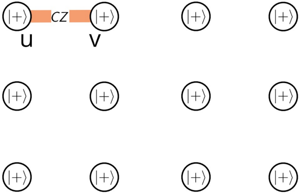
For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

A **quantum gate** is a matrix with $UU^\dagger = U^\dagger U = I$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$|G\rangle$

$$CZ_{uv} |+\rangle^{\otimes V} = CZ_{uv} \frac{1}{\sqrt{2}^n} \sum_{s \in \{0,1\}^V} |s\rangle$$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1 \\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0 \\ 1\end{smallmatrix}\right]$.

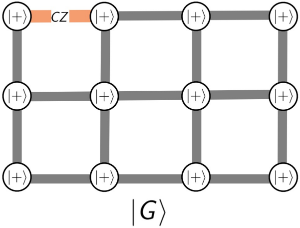
Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An **n-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

A **quantum gate** is a matrix with $UU^\dagger = U^\dagger U = I$.

The **controlled-Z** gate CZ_{uv} negates $|s\rangle$ if $s_u = s_v = 1$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$$\left(\prod_{uv\in E} CZ_{uv}\right)|+\rangle^{\otimes V}$$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right] \in \mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

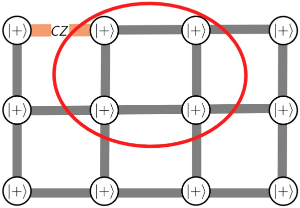
Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

A **quantum gate** is a matrix with $UU^\dagger = U^\dagger U = I$.

The **controlled-Z** gate CZ_{uv} negates $|s\rangle$ if $s_u = s_v = 1$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$$\left(\prod_{uv\in E} CZ_{uv}\right)|+\rangle^{\otimes V} = \frac{1}{\sqrt{2^n}}\sum_{s\in\{0,1\}^V}(-1)^{|E(supp_s)|}|s\rangle$$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\\beta\end{smallmatrix}\right]\in\mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$.

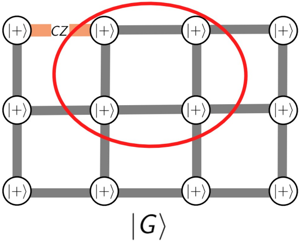
Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2\otimes\ldots\otimes\mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

A **quantum gate** is a matrix with $UU^\dagger = U^\dagger U = I$.

The **controlled-*Z*** gate CZ_{uv} negates $|s\rangle$ if $s_u = s_v = 1$.

What is the **quantum state** $|G\rangle$ associated with a graph G?



$$\left(\prod_{uv\in E} CZ_{uv}\right)|+\rangle^{\otimes V} = \frac{1}{\sqrt{2}^n}\sum_{s\in\{0,1\}^V}(-1)^{|E(supp_s)|}|s\rangle$$

A **1-qubit quantum state** is $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right]\in\mathbb{C}^2$ with norm 1.

For example, $|0\rangle = \left[\begin{smallmatrix}1\\ 0\end{smallmatrix}\right]$ and $|1\rangle = \left[\begin{smallmatrix}0\\ 1\end{smallmatrix}\right]$.

Each vertex is a qubit initialized to $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$.

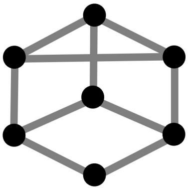
An ***n*-qubit quantum state** is $|\phi\rangle \in \mathbb{C}^2\otimes\ldots\otimes\mathbb{C}^2 = (\mathbb{C}^2)^{\otimes n}$.

A **quantum gate** is a matrix with $UU^\dagger = U^\dagger U = I$.

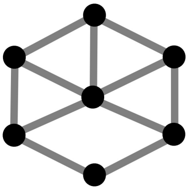
The **controlled-Z** gate CZ_{uv} negates $|s\rangle$ if $s_u = s_v = 1$.

A state is **entangled** if it *cannot* be written as $\bigotimes_{v\in V}|\phi_v\rangle$.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?

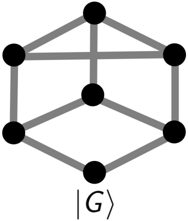


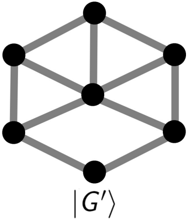
$|G\rangle$



$|G'\rangle$

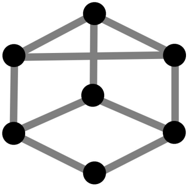
When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



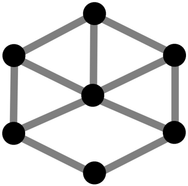


Two states on the same set of qubits are **local unitary equivalent** if they only differ up to 1-qubit gates.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$



$|G'\rangle$

Two states on the same set of qubits are **local unitary equivalent** if they only differ up to 1-qubit gates.

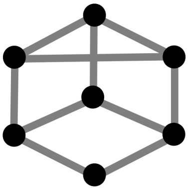
Example: We can switch 0s and 1s on the first qubit:

$$|000\rangle + |001\rangle + |111\rangle$$

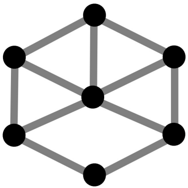
$$\simeq_{LU}$$

$$|100\rangle + |101\rangle + |011\rangle$$

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$

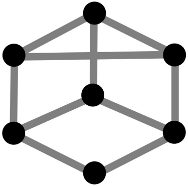


$|G'\rangle$

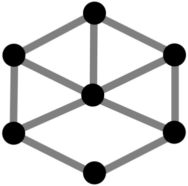
Two states on the same set of qubits are **local unitary equivalent** if they only differ up to 1-qubit gates.

Example: We can switch 0s and 1s on the first qubit; apply the gate $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to the first qubit.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$

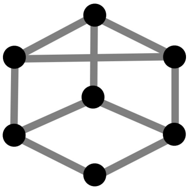


$|G'\rangle$

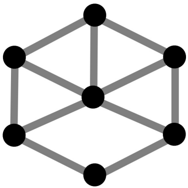
Two states on the same set of qubits are **local unitary equivalent** if they only differ up to 1-qubit gates.

Equivalent states have the same "level of entanglement" since $U_1 (|\phi_1\rangle \otimes |\phi_2\rangle \otimes \ldots \otimes |\phi_n\rangle) = (U_1 |\phi_1\rangle) \otimes |\phi_2\rangle \otimes \ldots \otimes |\phi_n\rangle$.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$



$|G'\rangle$

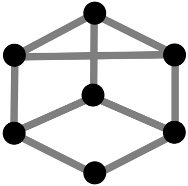
Two states on the same set of qubits are **local unitary equivalent** if they only differ up to 1-qubit gates.

Equivalent states have the same "level of entanglement" since $U_1(|\phi_1\rangle \otimes |\phi_2\rangle \otimes \ldots \otimes |\phi_n\rangle) = (U_1|\phi_1\rangle) \otimes |\phi_2\rangle \otimes \ldots \otimes |\phi_n\rangle$.

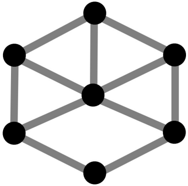
Theorem (Schlingemann 2002)

Every **stabilizer** *state is LU-equivalent to a graph state.*

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$

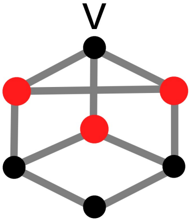


$|G'\rangle$

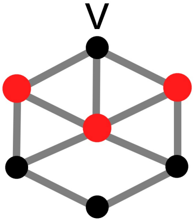
Two states on the same set of qubits are **local Clifford equivalent** if they only differ up to 1-qubit gates generated by H and S.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$



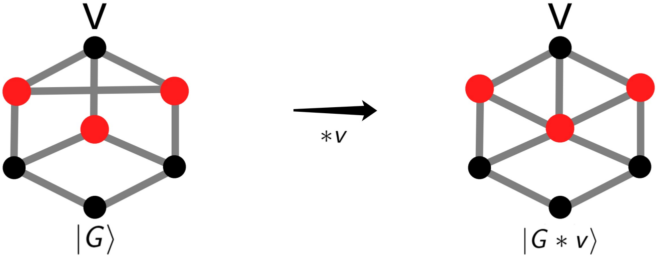
$|G'\rangle$

Two states on the same set of qubits are **local Clifford equivalent** if they only differ up to 1-qubit gates generated by H and S.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Given $v \in V$, perform HSH at v and $S^\dagger$ at each **neighbor** of v.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?

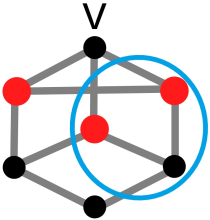


Two states on the same set of qubits are **local Clifford equivalent** if they only differ up to 1-qubit gates generated by H and S.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

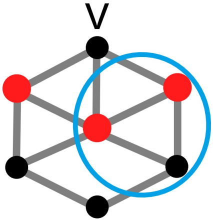
Given $v \in V$, perform HSH at v and $S^\dagger$ at each **neighbor** of v. The resulting graph $G * v$ is obtained from G by **locally complementing** at v: switching adjacencies within $N(v)$.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle$

$(-1)^{|E(supp_s)|}\,|s\rangle$
changes sign
iff
$|N(v) \cap supp_s|$
$\equiv 2, 3 \mod 4$



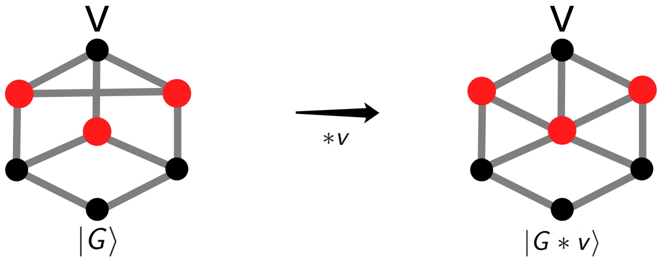
$|G * v\rangle$

Two states on the same set of qubits are **local Clifford equivalent** if they only differ up to 1-qubit gates generated by H and S.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

Given $v \in V$, perform HSH at v and $S^\dagger$ at each **neighbor** of v. The resulting graph $G * v$ is obtained from G by **locally complementing** at v: switching adjacencies within $N(v)$.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?

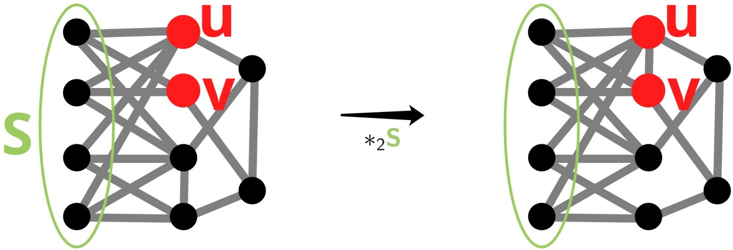


Theorem (Van den Nest, Dehaene, De Moor 2004)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local Clifford equivalent** *iff there is a sequence of vertices so that $|G'\rangle = |G * v_1 * \ldots * v_k\rangle$.*

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?

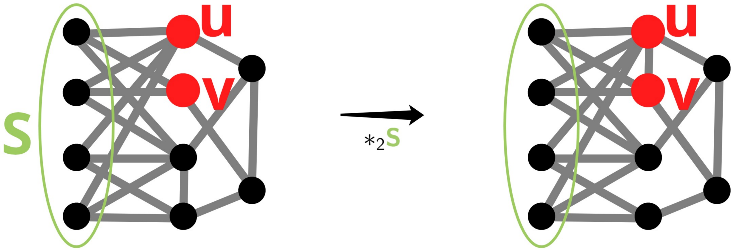


Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by* ***r*-local complementation**, *for $r \in \mathbb{Z}^+$.*

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \quad T = \begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix} \quad S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix} \quad \cdots$$

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?

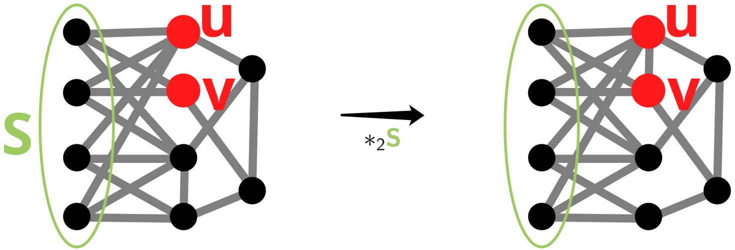


Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by r-***local complementation***, for $r \in \mathbb{Z}^+$.*

If S is an independent set and $G * S = G$, i.e., every pair u, v have $|N(u) \cap N(v) \cap S| \equiv 0 \mod 2$, then we may instead switch adjencies between u and v if $|N(u) \cap N(v) \cap S| \equiv 2 \mod 4$.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



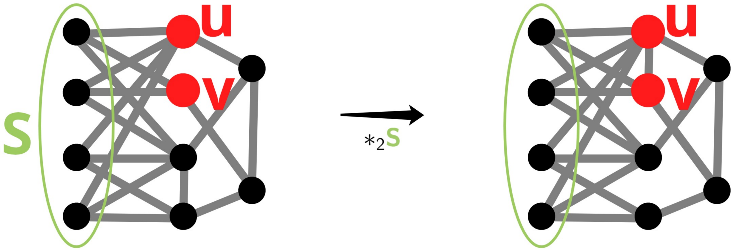
Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by* ***r*-local complementation**, *for $r \in \mathbb{Z}^+$.*

Meta Conjecture (originally Schlingemann; see Krueger & Werner 05)

In general, r-local complementation is "not much more powerful" than local complementation.

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



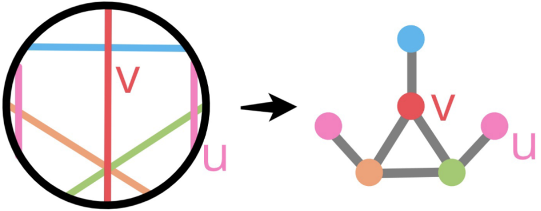
Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by* ***r*-local complementation***, for $r \in \mathbb{Z}^+$.*

Theorem (Ji, Chen, Wei, and Ying 2010)

There exists a pair of graph states which are LU-equivalent but **not** *LC-equivalent.*

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



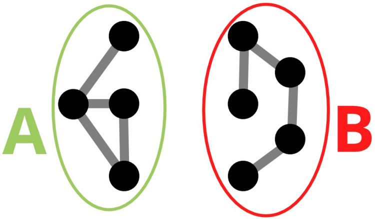
Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by* r-**local complementation**, *for $r \in \mathbb{Z}^+$.*

Theorem (Claudet, Geelen, Hahn, McCarty, Poulsen 2025+)

For **circle graphs**, *LU-equivalence $\leftrightarrow$ LC-equivalence.*

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



$|G\rangle = |\phi^{A}\rangle \otimes |\phi^{B}\rangle$

cut-rank$(A, B) = 0$

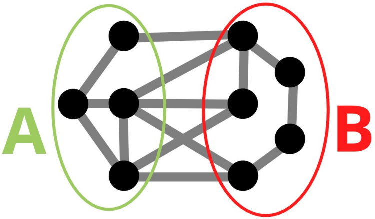
Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by r-***local complementation***, for $r \in \mathbb{Z}^{+}$.*

Lemma (Hein, Eisert, and Briegel 2004)

LU-equivalent graphs have the same **cut-rank** *function.*

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



While minimizing r, write:

$$|G\rangle = \sum_{i=1}^{r} \xi_i \, |\phi_i^{A}\rangle \otimes |\phi_i^{B}\rangle.$$

cut-rank(A, B) $= \log_2(r)$

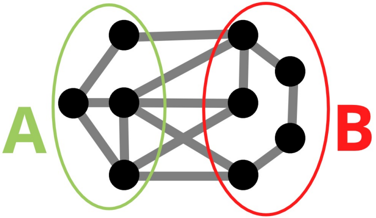
Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by r-***local complementation***, for $r \in \mathbb{Z}^+$.*

Lemma (Hein, Eisert, and Briegel 2004)

LU-equivalent graphs have the same **cut-rank** *function.*

When are two graph states $|G\rangle$ and $|G'\rangle$ **equivalent**?



While minimizing r, write:

$$|G\rangle = \sum_{i=1}^{r} \xi_i \, |\phi_i^{A}\rangle \otimes |\phi_i^{B}\rangle.$$

$$\textbf{cut-rank}(A, B) = \log_2(r)$$
$$= \text{rank}_{GF(2)}(Adj[A, B])$$

Theorem (Claudet and Perdrix 2025)

Two graph states $|G\rangle$ and $|G'\rangle$ are **local unitary equivalent** *iff G' can be obtained from G by* r-**local complementation**, *for $r \in \mathbb{Z}^+$.*

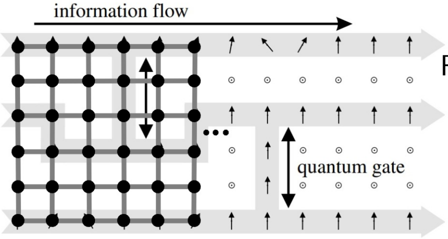
Lemma (Hein, Eisert, and Briegel 2004)

LU-equivalent graphs have the same **cut-rank** *function.*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"

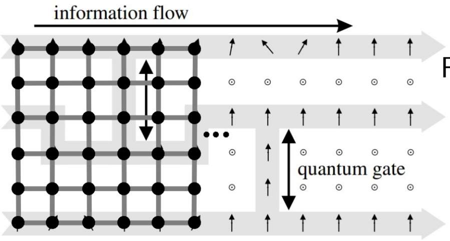


Raussendorf & Briegel 2001

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



Raussendorf & Briegel 2001

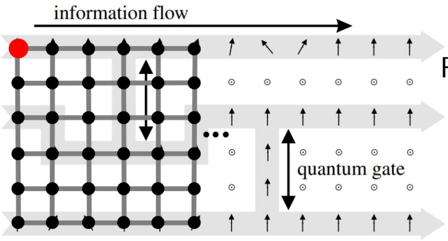
In **measurement-based quantum computation**, we:

- First prepare a graph state $|G\rangle$.

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



Raussendorf & Briegel 2001

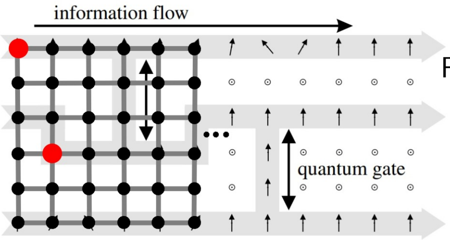
In **measurement-based quantum computation**, we:

- First prepare a graph state $|G\rangle$.
- Measure a qubit: measuring $\left[\begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}\right]$ in the computational basis returns 0 with probability $|\alpha|^2$ and 1 with probability $|\beta|^2$.

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



Raussendorf & Briegel 2001

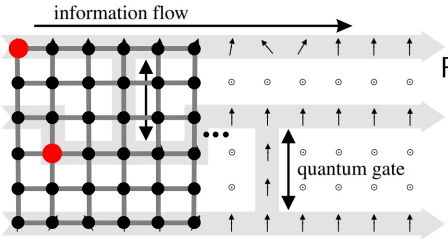
In **measurement-based quantum computation**, we:

- First prepare a graph state $|G\rangle$.
- Measure a qubit: measuring $\left[\begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}\right]$ in the computational basis returns 0 with probability $|\alpha|^2$ and 1 with probability $|\beta|^2$.
- Choose the next measurement based on prior outcomes.

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



Raussendorf & Briegel 2001

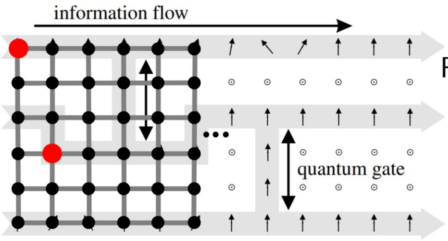
Theorem (Raussendorf and Briegel 2001)

When every **grid** *("2D cluster state") can be prepared, this is equivalent to the quantum gate model (up to polynomial factors).*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



Raussendorf & Briegel 2001

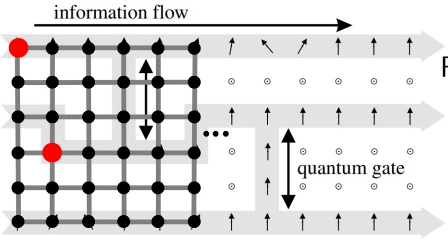
Question

Are there graphs which can **more efficiently** *model every n-qubit quantum gate than* **grids***?*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

"A One-Way Quantum Computer"



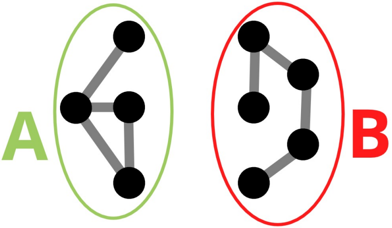
Raussendorf & Briegel 2001

Question (see Rossi, Huber, Bruß, & Macciavello 13)

Are there architectures which are easier to build experimentally? Perhaps **hypergraph states***?*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

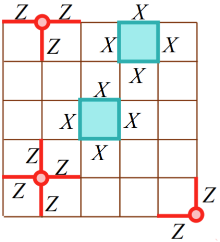


Theorem (Van den Nest, Dür, Vidal, & Briegel 2007)

Classes of graphs with logarithmic **rank-width** *(i.e. "low entanglement") only yield classical computers.*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*



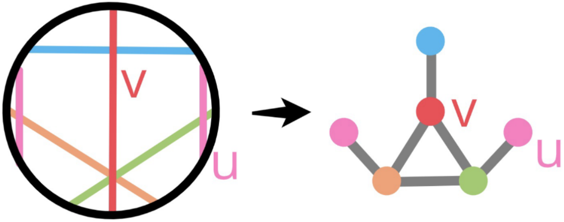
$\longrightarrow$ **stabilizer state**

Theorem (Bravyi & Raussendorf 07 + Bravyi, Gosset, & Liu 22)

However, high entanglement is not sufficient; there are also "topological/geometric" obstructions.

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

Which graph classes can serve as **universal resources** *for quantum computation?*

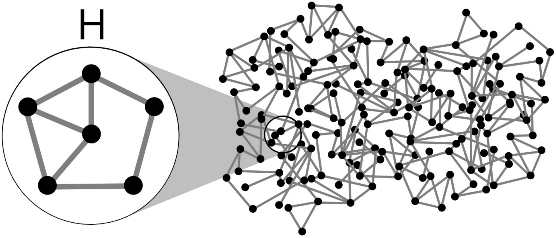


Theorem (Harrison, Iyer, Parekh, Thompson, Zhao 25+)

The class of all **circle graphs** *also yields a classical computer.*

Question (see Van den Nest, Dür, Vidal, & Briegel 2007)

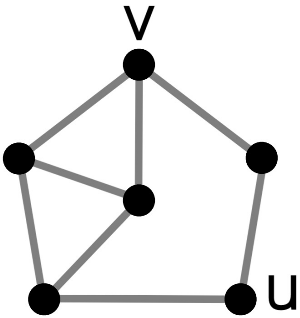
Which graph classes can serve as **universal resources** *for quantum computation?*



Conjecture (Geelen)

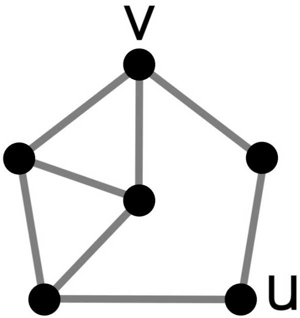
Every class that does **not** *yield a classical computer* **contains all graphs** *up to local complementation and vertex deletion.*

The **vertex-minors** of a graph G are the graphs that can be obtained from G by:



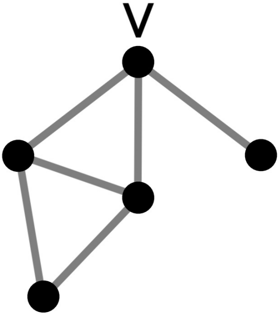
The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices



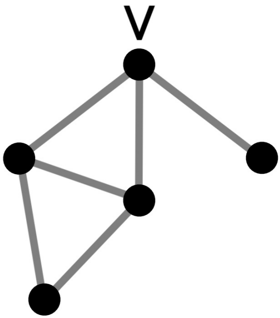
The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices



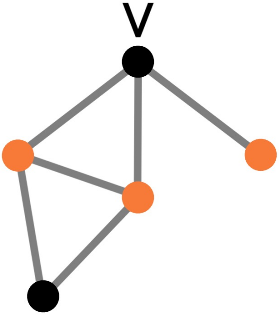
The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices



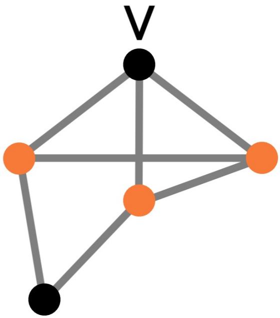
The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

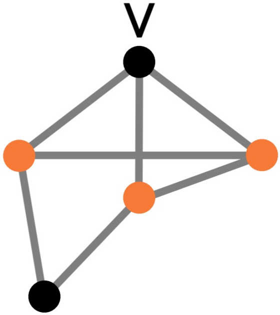
- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

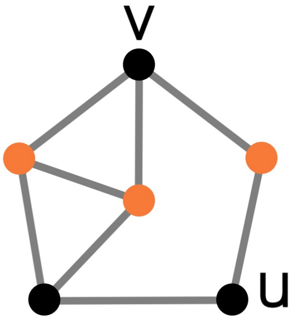
We may do all local complementations first.



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

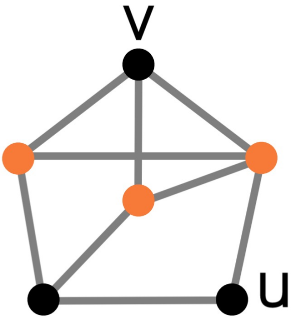
We may do all local complementations first.



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

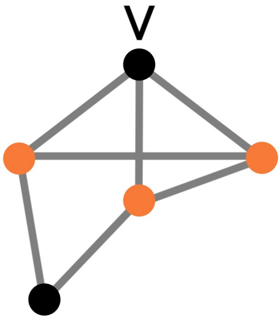
We may do all local complementations first.



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

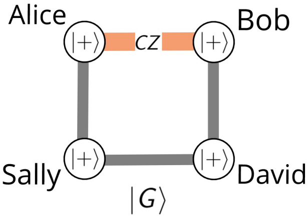
We may do all local complementations first.



The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

We may do all local complementations first.



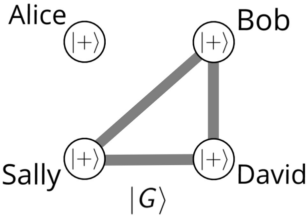
Proposition (Dahlberg-Helsen-Wehner 20)

A graph H without isolated vertices is a **vertex-minor** *of G iff $|H\rangle$ can be prepared from $|G\rangle$ using $LC + LPM + CC$.*

The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

We may do all local complementations first.



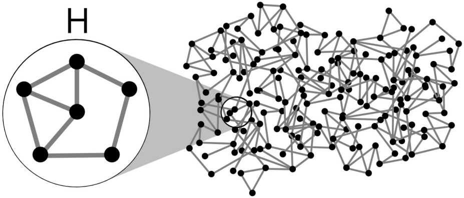
Proposition (Dahlberg-Helsen-Wehner 20)

A graph H without isolated vertices is a **vertex-minor** *of G iff $|H\rangle$ can be prepared from $|G\rangle$ using LC + LPM + CC.*

The **vertex-minors** of a graph G are the graphs that can be obtained from G by:

- deleting vertices and
- **locally complementing** at vertices (replaces the induced subgraph on the neighborhood of v by its complement).

We may do all local complementations first.



Theorem (Cautrès, Claudet, Mhalla, Perdrix, Savin, & Thomassé 2024)

There are graphs with $\mathcal{O}(n^2)$-many vertices which ***contain every n-vertex*** *graph as a vertex-minor. This is best possible.*

Structure Theorem (Robertson & Seymour 2003)

The graphs in any proper **minor-closed** *class "decompose" into parts that "almost embed" in a surface of bounded genus.*

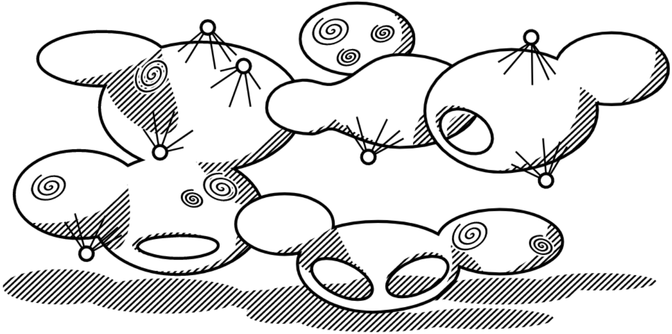
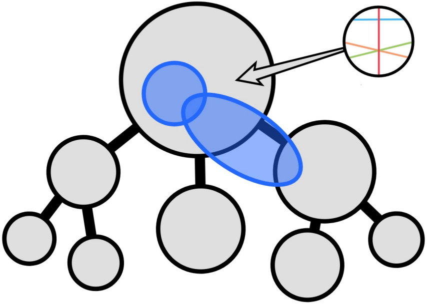


Figure by Felix Reidl

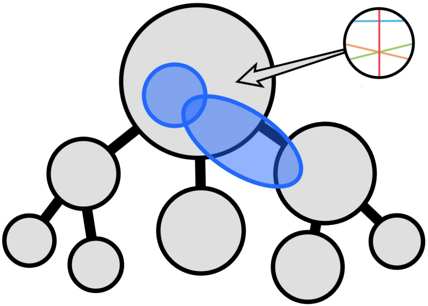
Conjecture (Geelen)

The graphs in any proper **vertex-minor***-closed class "decompose" into parts that are "almost"* **circle graphs***.*



Conjecture (Geelen)

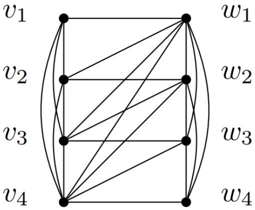
The graphs in any proper **vertex-minor***-closed class "decompose" into parts that are "almost"* **circle graphs***.*



Ongoing project with Jim Geelen & Paul Wollan aiming to prove the conjecture.

Theorem (Kwon, McCarty, Oum, Wollan 2021)

A graph class "looks like shallow trees" (w.r.t. **cut-rank***) iff it does not contain all* **half-graphs** *as vertex-minors.*



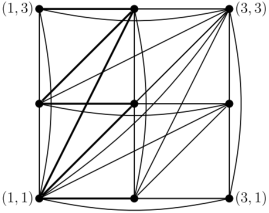
H

Theorem (Kwon, McCarty, Oum, Wollan 2021)

A graph class "looks like shallow trees" (w.r.t. **cut-rank***) iff it does not contain all* **half-graphs** *as vertex-minors.*

Theorem (Geelen, Kwon, McCarty, Wollan 2023)

A graph class "looks like trees" (w.r.t. **cut-rank***) iff it does not contain all* **comparability grids** *as vertex-minors.*



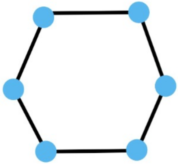
H

Theorem (Kwon, McCarty, Oum, Wollan 2021)

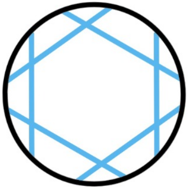
A graph class "looks like shallow trees" (w.r.t. **cut-rank***) iff it does not contain all* **half-graphs** *as vertex-minors.*

Theorem (Geelen, Kwon, McCarty, Wollan 2023)

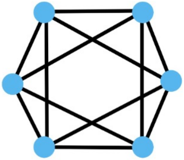
A graph class "looks like trees" (w.r.t. **cut-rank***) iff it does not contain all* **comparability grids** *as vertex-minors.*



circle graph



chord diagram

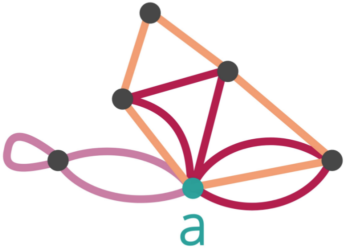


tour graph

Theorem (McCarty 24)

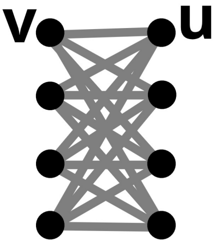
*For any Eulerian graph G and vertex **a**, the **maximum** size of a circuit decomposition where every circuit is odd and hits **a** equals*

$$\text{minimum}_{\gamma', X}\left(\gamma'(E(X)) + \frac{1}{2}|\delta(X)| - odd_{\gamma'}(G - X)\right).$$



- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?

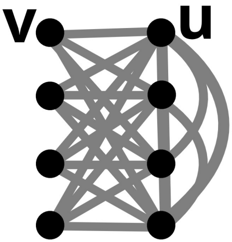
$$\text{Apply} \prod_{xy \in E} CZ_{xy}$$



G

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?

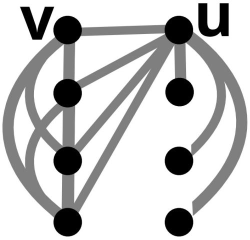
Apply $\prod_{xy \in E} CZ_{xy}$



$G * v$

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?

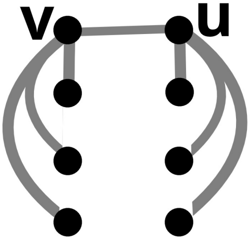
Apply $\prod_{xy \in E} CZ_{xy}$



$G * v * u$

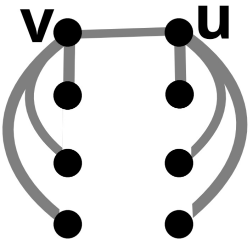
- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?

Apply $\prod_{xy \in E} CZ_{xy}$

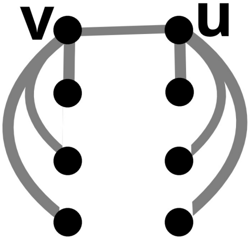


$G * v * u * v$

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).

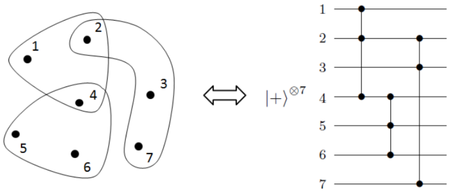


- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).

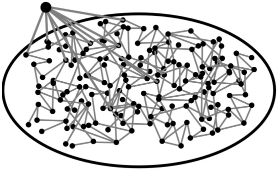


- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).
- **Q**: **Hypergraph** states? (Rossi, Huber, Bruß, Macciavello 13)



- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).
- **Q**: **Hypergraph** states? (Rossi, Huber, Bruß, Macciavello 13)
- **Q**: If an adversary secretly entangled a few qubits with $|G\rangle$, how badly can they affect $|G\rangle$?



- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).
- **Q**: **Hypergraph** states? (Rossi, Huber, Bruß, Macciavello 13)
- **Q**: If an adversary secretly entangled a few qubits with $|G\rangle$, how badly can they affect $|G\rangle$?
  - Can this kill a quantum computer?

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).
- **Q**: **Hypergraph** states? (Rossi, Huber, Bruß, Macciavello 13)
- **Q**: If an adversary secretly entangled a few qubits with $|G\rangle$, how badly can they affect $|G\rangle$?
  - Can this kill a quantum computer?
  - Algorithms to discover vulnerabilities? (See Fomin, Golovach, Strømme, Thilikos 2018. Working with Sang-Yoon Kim.)

- **Q**: How difficult is it to **prepare** a desired graph state $|G\rangle$?
  - Algorithmic aspects of finding a **locally equivalent** graph G' with as few edges as possible (Sharma, Goodenough, Borregaard, Rozpędek, Helsen, 2025).
  - Going back and forth between local complementations and edge deletions; **CZ-distance** (see Davies & Jena 2025).
  - When can we locally complement to **remove big bicliques**? Related to (Blanco 2023) and (Brianski, Koutecky, Kral, Pekarkova, Schroder 2024). This implies sparsity or an H-vertex-minor by (Campbell, Davies, Hickingbotham 2025).
- **Q**: **Hypergraph** states? (Rossi, Huber, Bruß, Macciavello 13)
- **Q**: If an adversary secretly entangled a few qubits with $|G\rangle$, how badly can they affect $|G\rangle$?
  - Can this kill a quantum computer?
  - Algorithms to discover vulnerabilities? (See Fomin, Golovach, Strømme, Thilikos 2018. Working with Sang-Yoon Kim.)
- **Q**: Ask Caleb about random graphs.

Thank you!