Monadically stable graph classes

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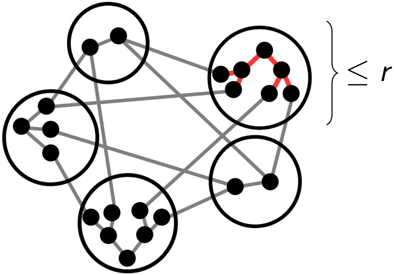
Oberwolfach Graph Theory Workshop

Theorem ("Sparsity", Nešetřil & Ossona de Mendez 2012)

The following are equivalent for any graph class $\mathcal{F}$.

1) *For each r, $\mathcal{F}$ forbids some graph as an* ***r*-shallow minor**.



A minor is ***r*-shallow** if each bag has radius $\leq r$.
A class is **nowhere dense** if it forbids some graph as an r-shallow minor for each $r \in \mathbb{N}$.

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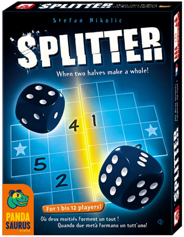

The **radius-r splitter game** is a pursuit-evasion game where the “robber” moves $\leq r$ steps per “cop” deployment. **Splitter wins** by catching the robber with $\leq f(r)$ deployments.

Theorem (Adler & Adler 2014)

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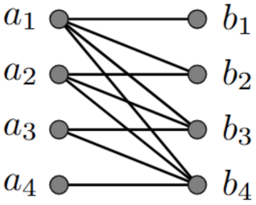

Roughly, a class $\mathcal{F}$ is **stable** if first-order logic cannot define all **half-graphs** H_n on $\mathcal{F}$.

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If ϕ is true for G, we say G **models** ϕ and write $G \models \phi$.

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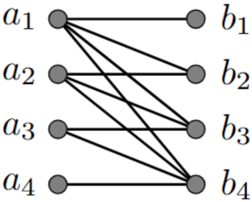

A class $\mathcal{F}$ is **stable** if there is no formula $\phi(a, b)$ st for all n, there is a graph $G \in \mathcal{F}$ with tuples $\bar{a}_1, \ldots, \bar{a}_n$ and $\bar{b}_1, \ldots, \bar{b}_n$ st the pairs $\bar{a}_i, \bar{b}_j$ with $G \models \phi(\bar{a}_i, \bar{b}_j)$ form the **half-graph** H_n.

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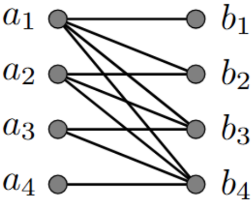

Monadically stable classes are the same as **stable** classes, under the assumption that the class is closed under taking induced subgraphs (Braunfeld & Laskowski 2022).

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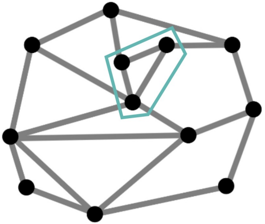

FO model-checking is FPT: Can determine if an n-vertex $G \in \mathcal{F}$ satisfies a fixed first-order property ϕ in time $\mathcal{O}_\phi(n^{c_\mathcal{F}})$.

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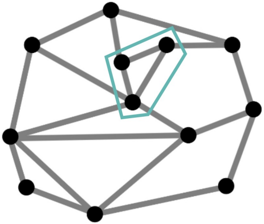

For instance, on the classes above, we can determine if H is a subgraph of an n-vertex $G \in \mathcal{F}$ in time $\mathcal{O}_H(n^{1.0001})$.

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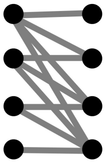

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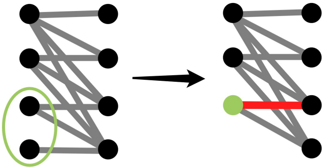

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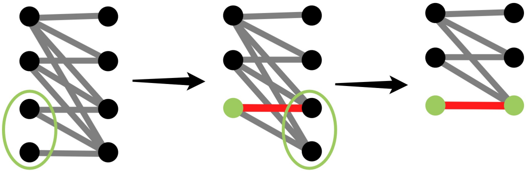

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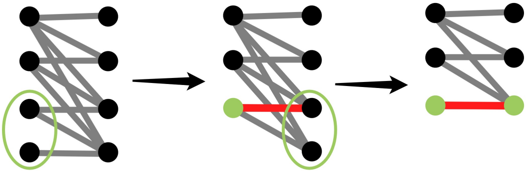

Moral: Analogous items known if we "forbid" a half-graph!

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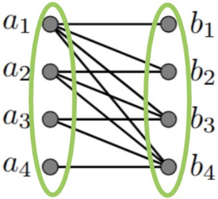

A class $\mathcal{F}$ is **edge-stable** if there exists n so that no $G \in \mathcal{F}$ has the **half-graph** H_n induced *between* its sides **X** and **Y**.

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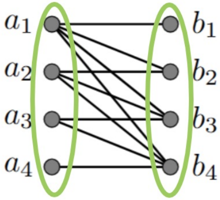

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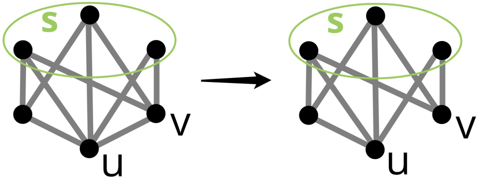

An r-**shallow vertex-minor** is obtained by, $\leq r$ times, **locally complementing** on an independent set S. This switches adjacency between $u, v \notin S$ if $|(N(u) \cap N(v)) \cap S|$ is odd.

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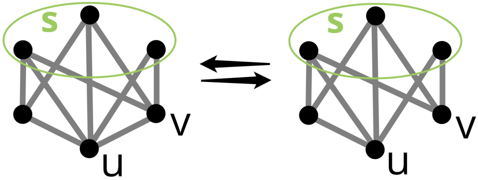

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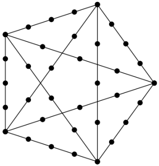

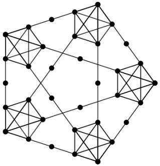

There is also a characterization by forbidding **induced subgraphs** which are "nearly" $\leq r$-subdivisions of cliques, or the line graphs of $\leq r$-subdivisions of cliques.

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1) *For each* r, $\mathcal{F}$ *forbids* $H_{f(r)}$ *as* **r-shallow vertex-minor**.
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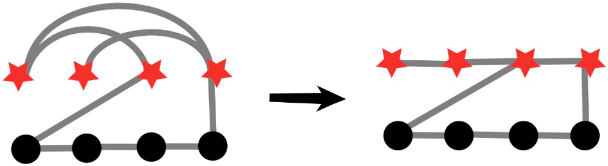

The **Flipper game** is like the Splitter game, except instead of deploying cops, the Flipper player can "flip". To **flip**, select $X \subseteq V(G)$ and replace $G[X]$ by its complement.

The **radius-r Flipper game** is played by two players: Flipper and Connector. In each round:

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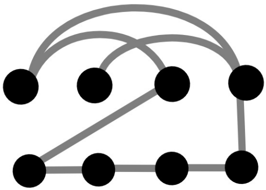

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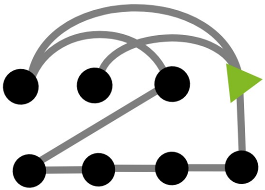

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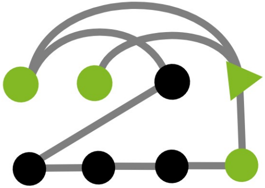

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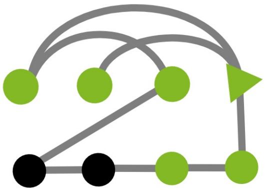

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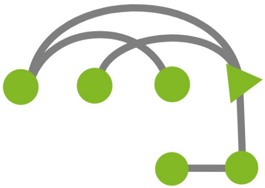

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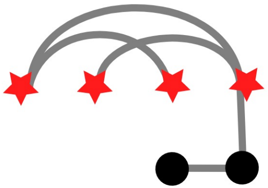

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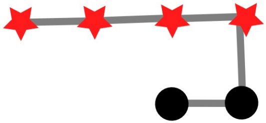

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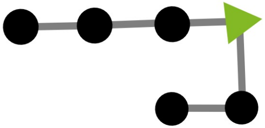

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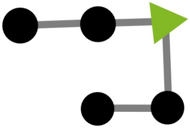

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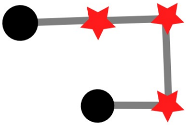

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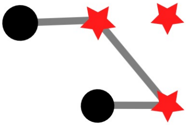

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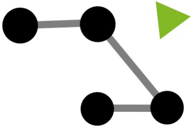

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Example with $r = 2$:

Flipper **wins** on a class $\mathcal{F}$ if there exists $t = t(r)$ so that Flipper wins in $\leq t$ rounds on each $G \in \mathcal{F}$.

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$\subseteq$

bounded expansion

(shallow minors have bounded average degree)

$\subseteq$

nowhere dense

(have forbidden shallow minors)

$\subseteq$

monadically stable

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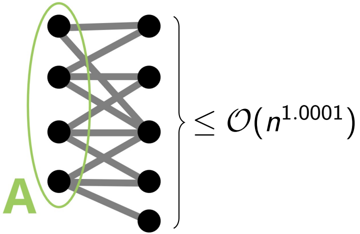

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Conjecture (Ossona de Mendez 2021)

A hereditary class is **stable** *if and only if it can be obtained from a* **nowhere dense** *class "using" first-order logic.*

Thank you!