Average degree and girth

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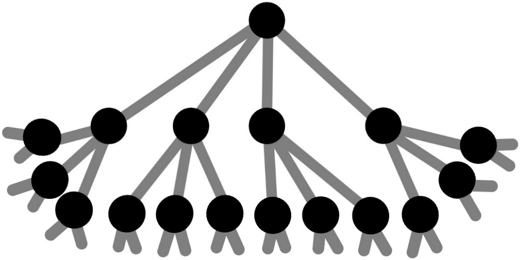
Theorem (Erdös 1959)

For any $d, k \in \mathbb{N}$, there exists a graph with avgdeg $\geq d$ and girth $\geq k$.



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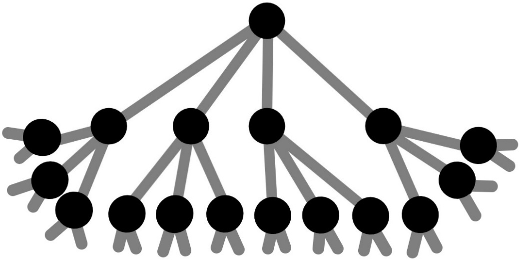

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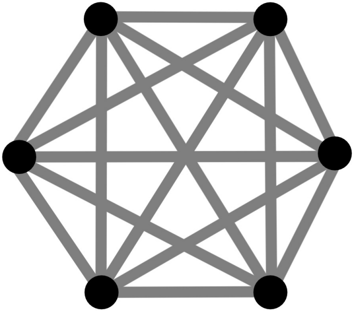

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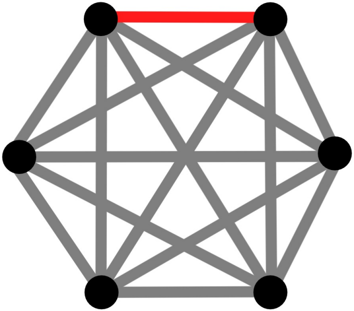

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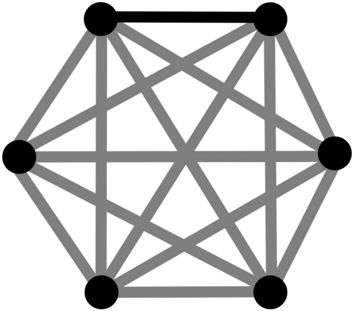

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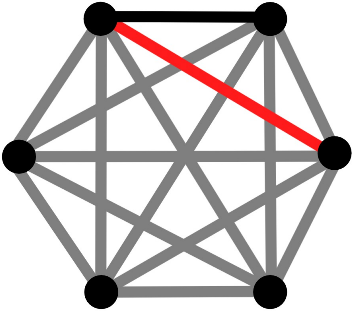

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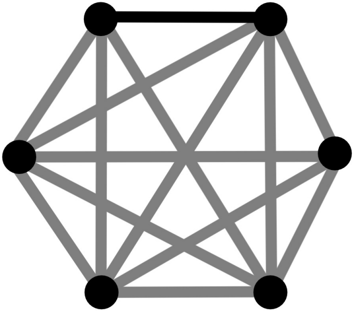

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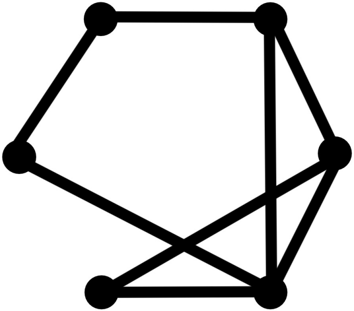

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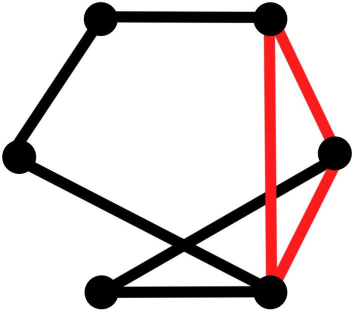

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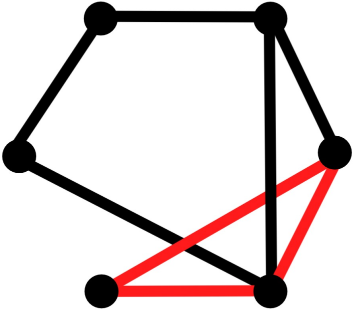

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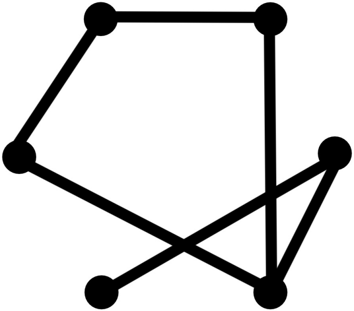

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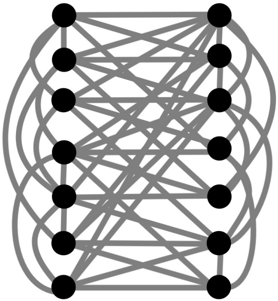
Conjecture (Thomassen 1983)

*There exists a function f so that **every** graph of avgdeg $\geq f(d, k)$ has a subgraph with avgdeg $\geq d$ and girth $\geq k$.*



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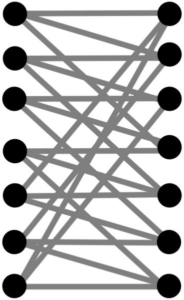

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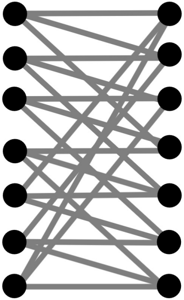

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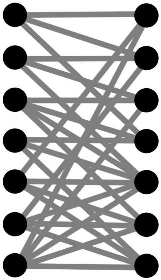
Theorem (Kühn-Osthus 2004)

Every ***bipartite*** *graph of avgdeg* $\geq f(d)$ *has a subgraph with avgdeg* $\geq d$ *and* ***no*** 4-***cycles***.

Conjecture

*Every graph of avgdeg $\geq f(t,d,k)$ has either a $K_{t,t}$-**subgraph** or an **induced** subgraph of avgdeg $\geq d$ and girth $\geq k$.*



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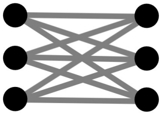

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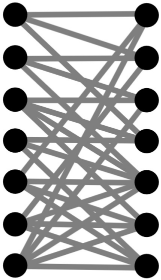

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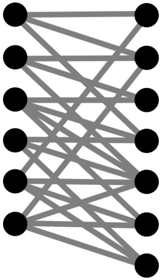

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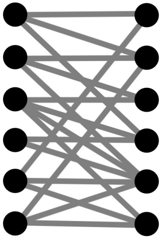

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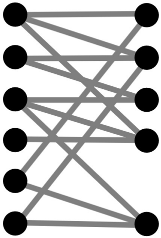

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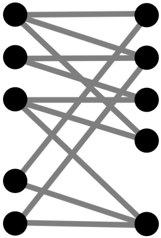

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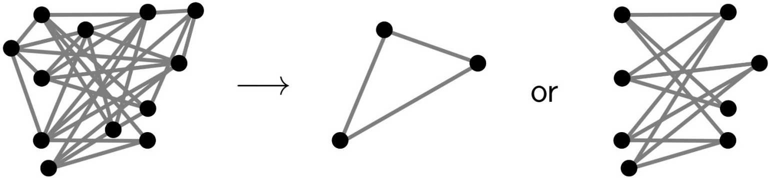

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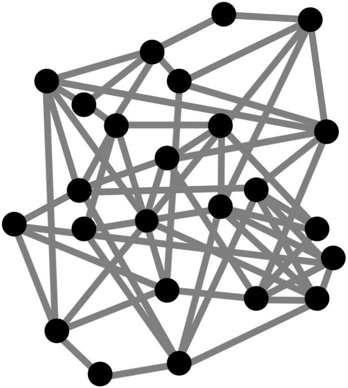
Theorem (2021)

*Every **bipartite** graph of avgdeg $\geq f(t,d)$ has either $K_{t,t}$ or an induced subgraph with avgdeg $\geq d$ and **no** 4-**cycles**.*

When is the maximum average degree of a graph tied to the size of its largest balanced biclique?



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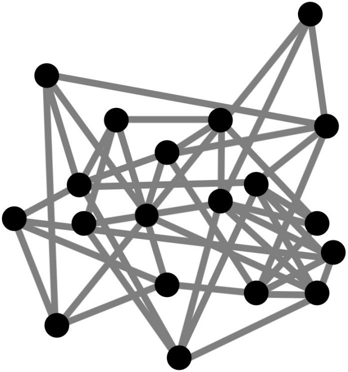

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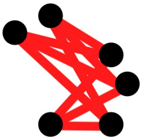

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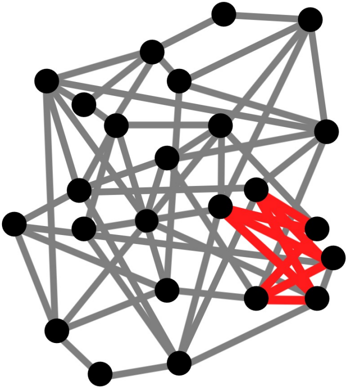
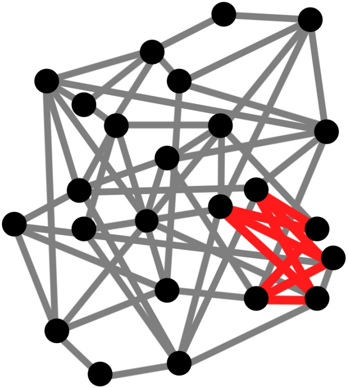


biclique number $\tau(G) :=$

maximum t so that G has $K_{t,t}$-subgraph

For which classes of graphs does there exist a function f so that **mad**$(G) \leq f(\tau(G))$?



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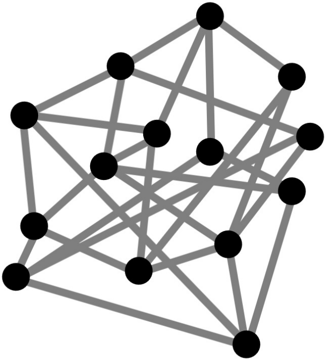

There exist graphs of arbitrarily large average degree and girth (Erdös).

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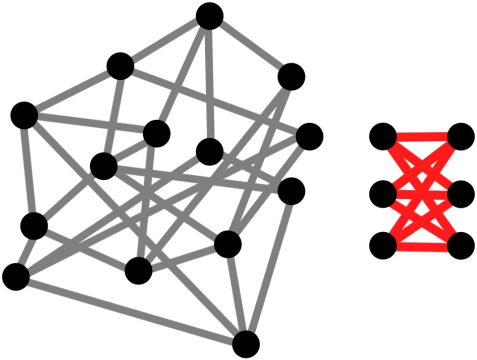

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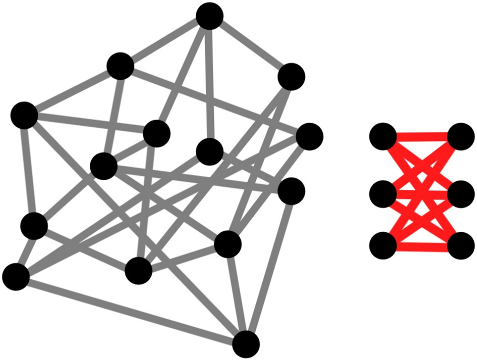

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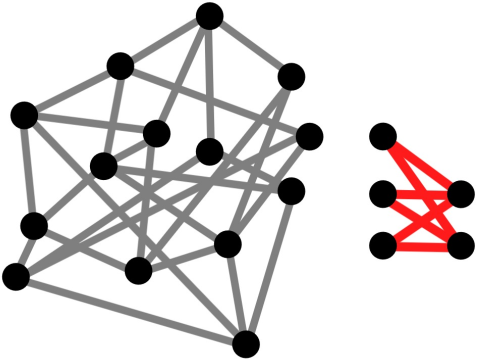

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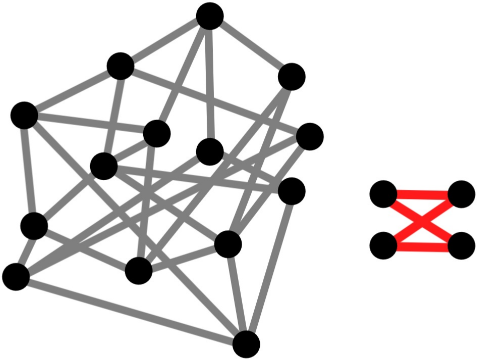

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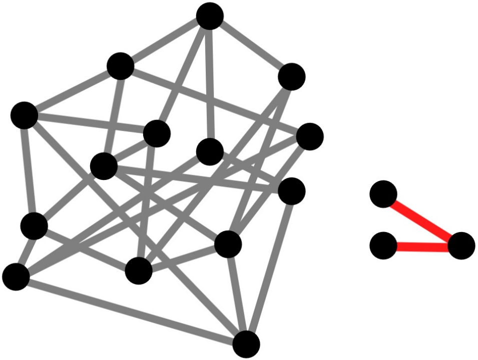

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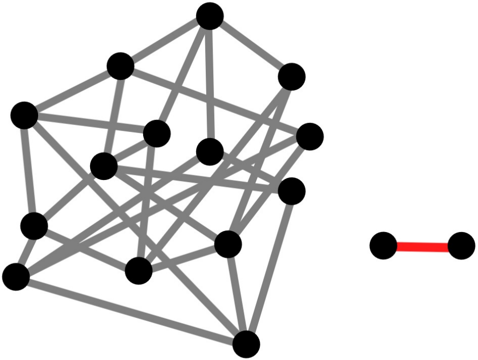

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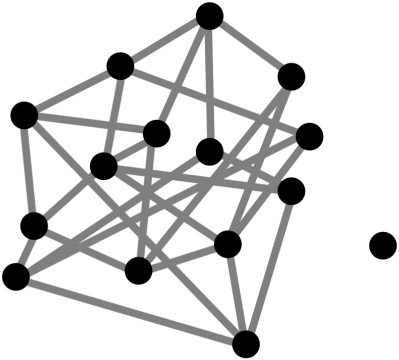

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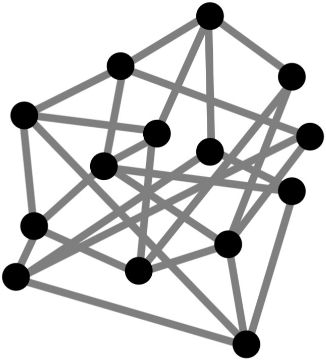

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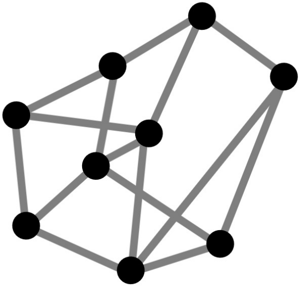

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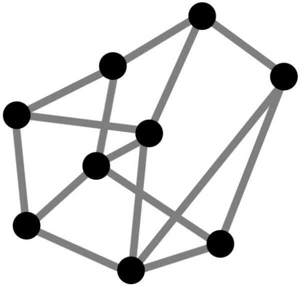

Call such a class **degree-bounded**
and f a **bounding function**.

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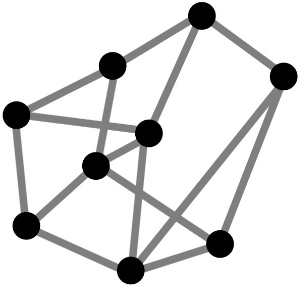

Conjecture

A class is **degree-bounded** $\Leftrightarrow$ *it does not contain graphs of arbitrarily large average degree and girth.*

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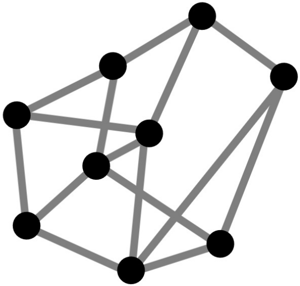

Theorem (Uses the theorem of Kwan, Letzter, Sudakov, Tran)

A class is **degree-bounded** $\Leftrightarrow$ *there exists* $c \in \mathbb{N}$ *such that every bipartite, 4-cycle-free graph in the class has avgdeg* $\leq c$*.*

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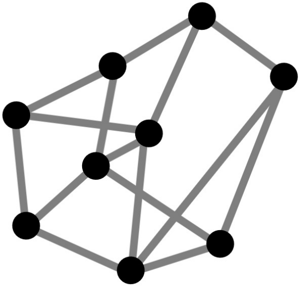

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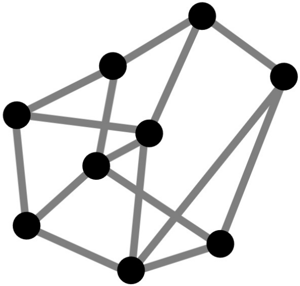

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Question

Does every degree-bounded class have a bounding function that is a ***polynomial****?*

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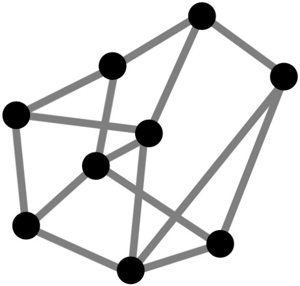

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Analogous to problems on **chromatic number**.

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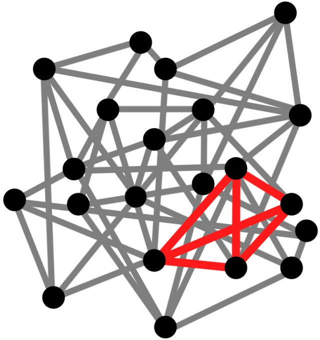

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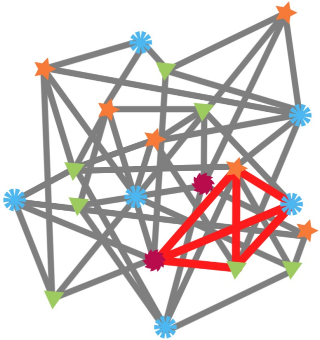

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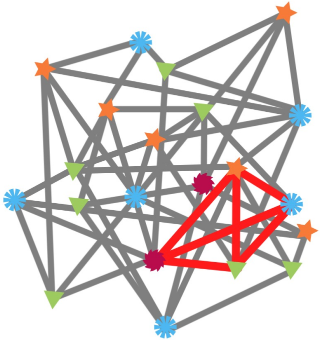

Call such a class χ-**bounded**
and f a χ-**bounding function**.

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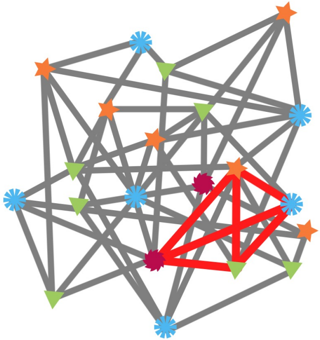

Theorem (Carbonero-Hompe-Moore-Spirkl 2022)

There exist graphs with arbitrarily large chromatic number but no triangle-free induced subgraph of chromatic number $\geq k$.

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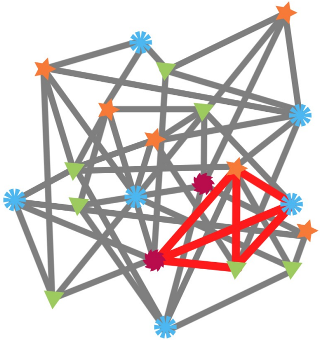

Theorem (Briański-Davies-Walczak 2022)

There is a χ-bounded class that has no polynomial χ-bounding function.

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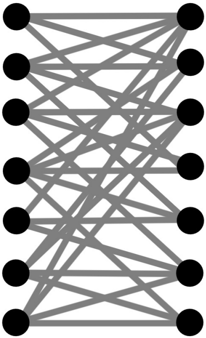

Not every χ-**bounded** class is **degree-bounded**.

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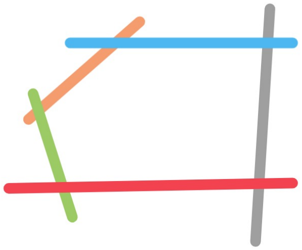

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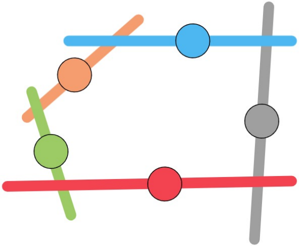

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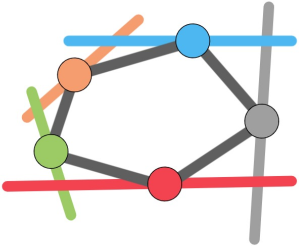

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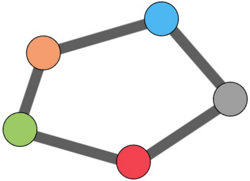

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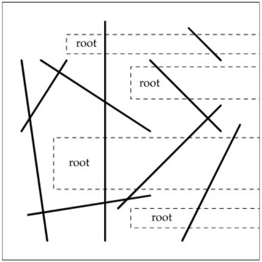

Fig. 1. Segments, probes and roots.

Pawlik, Kozik, Krawczyk Lasoń, Micek, Trotter, & Walczak showed that they are not χ-bounded.

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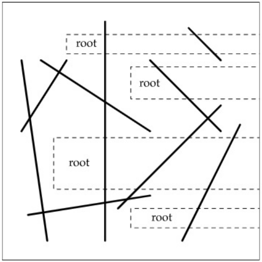

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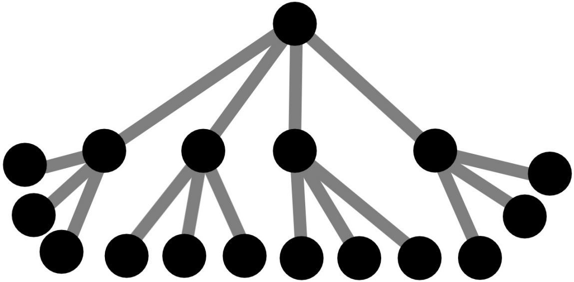
All of their induced subgraphs with **girth** ≥ 5 have bounded average degree (Fox-Pach 2008).

Theorem (Scott, Seymour, & Spirkl)

*For any forest F, the class of graphs with no induced F has avgdeg $\leq$ **poly**(τ).*



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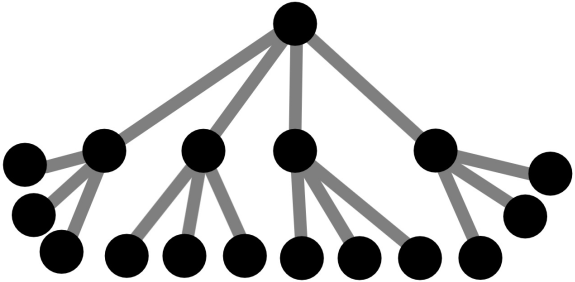

It is not known if these classes are χ-**bounded**;
this is the Gyárfás–Sumner Conjecture.

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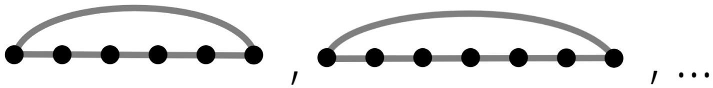

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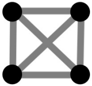

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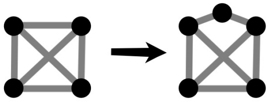

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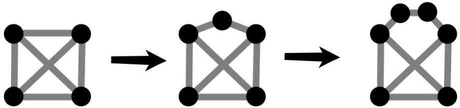

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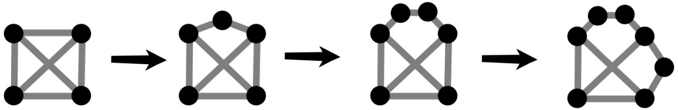

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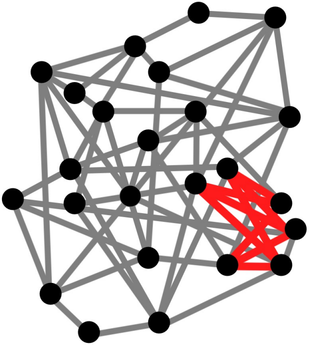

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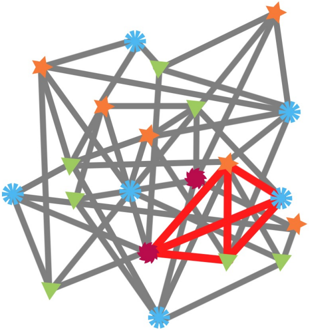

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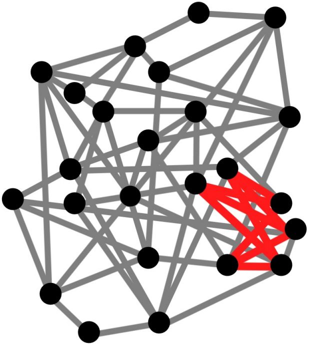

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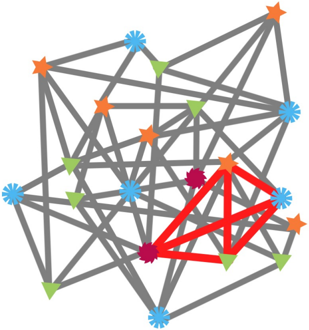

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Obtaining/improving the bound $\mathrm{avgdeg} \leq 2^{2^{2^{2^{poly(\tau)}}}}$

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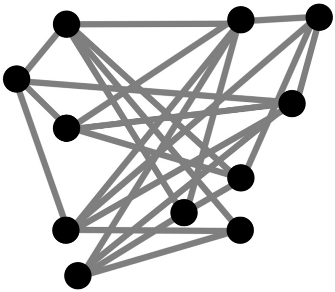
Every graph of avgdeg $\geq 2^{d^2 2^{\mathbf{poly(t)}}}$ has either K_t or an induced bipartite subgraph with avgdeg $\geq d$.



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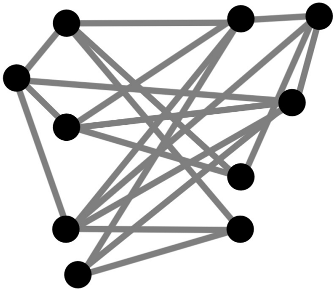

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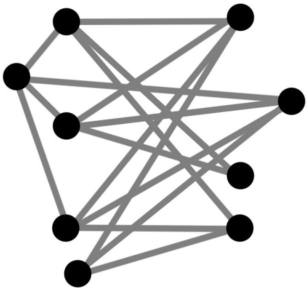

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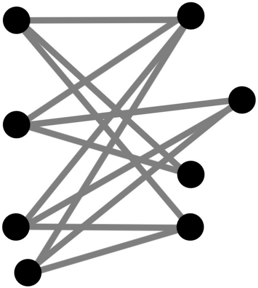

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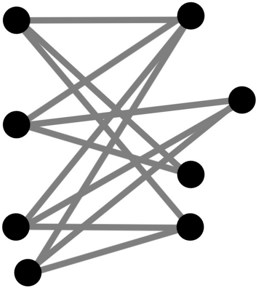
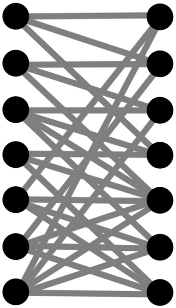


The function must be exponential in d.

Obtaining/improving the bound $\mathrm{avgdeg} \leq 2^{2^{2^{2^{poly(\tau)}}}}$

Theorem (Kühn & Osthus, 04)

Every bipartite graph of avgdeg $\geq 2^{2^{poly(d)}}$ has a subgraph with avgdeg $\geq d$ and no 4-cycles.



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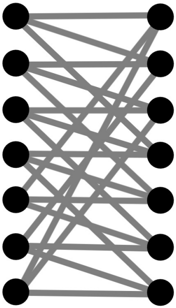

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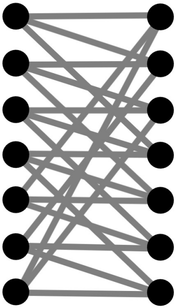

Showed a lower bound of $d^{3-o(1)}$.

Obtaining/improving the bound $\mathrm{avgdeg} \le 2^{2^{2^{2^{poly(\tau)}}}}$

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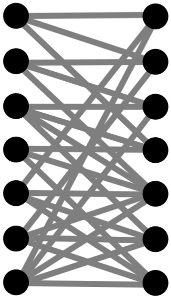

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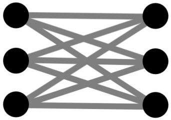

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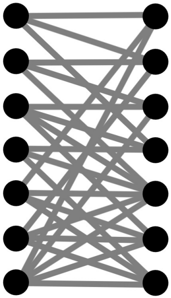

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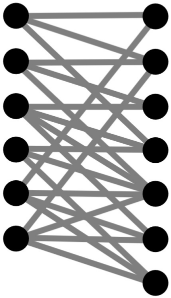

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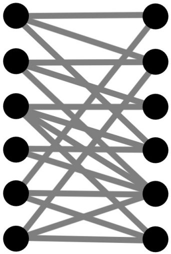

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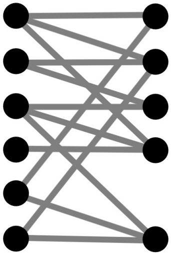

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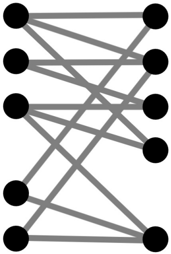

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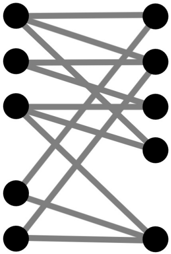

Based on a proof of Dellamonica, Koubek, Martin, & Rödl, 11.

Obtaining/improving the bound $\mathrm{avgdeg} \leq 2^{2^{2^{2^{poly(\tau)}}}}$

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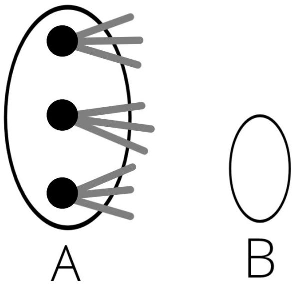

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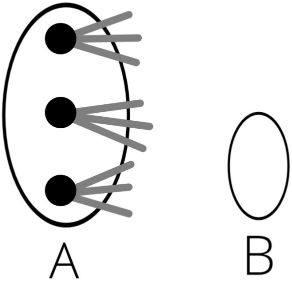

This lets us apply (Füredi, 83).

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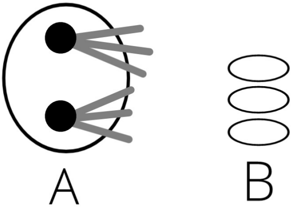

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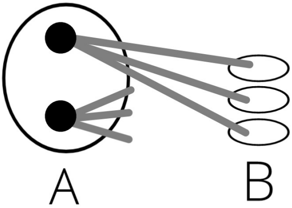

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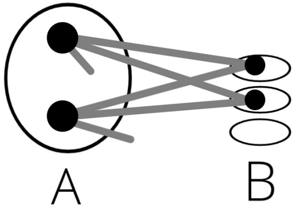

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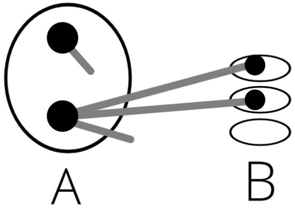

This lets us apply (Füredi, 83).

Obtaining/improving the bound $\mathrm{avgdeg} \leq 2^{2^{2^{2^{poly(\tau)}}}}$

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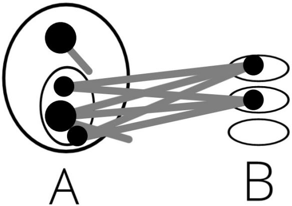

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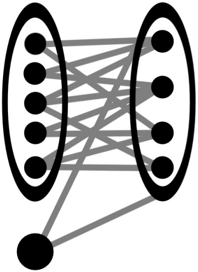

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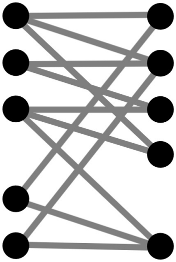

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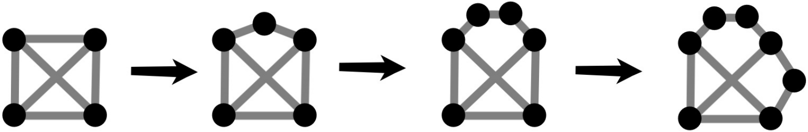
Question

Does every degree-bounded class have a bounding function that is a **polynomial***?*



Thank you!